A three dimensional electromagnetic shell finite element for coupled vector-scalar potential formulations

P. Thomas[1,2], Y Le Menach[1,3]
[1] LAMEL (Laboratoire d'Analyse des Matériels Electriques)
[2] Electricité de France, 1, avenue du Général De Gaulle, 92141, Clamart France
[3] Université Lille1, L2EP, Cité scientifique, 59655 Villeneuve d'ascq
pierre.thomas@edf.fr

Abstract —In many eddy current problems solved by FEM, meshing of the skin depth leads to CAD difficulties and heavy computational effort. That can be avoided using dedicated elements such surface impedances for perfect conductors, or shell elements for arbitrary electromagnetic medium. We present the basic principles of a shell element which is suitable for harmonic response and fully compatible with the electric and magnetic formulations used in *Code_Carmel3D*. We conclude with a first elementary test from the NDT domain, dealing with the effect of a thin copper deposit at the surface of a steel tube.

I. Introduction

Shell elements have been implemented in industrial and commercial electromagnetic codes since the middle of 90's [1], mainly dedicated to shielding applications. Since, much work has been done to take into account for the time domain or integrate it with other formulation [3], [4]. Since 2000, the electric and magnetic 3D formulations in terms of potential used by *Code_Carmel3D* [2] of L2EP (Laboratoire d'Electronique et d'Electrotechnique de Lille) and EDF covers a large scope of non-linear, circuit-coupled static or rotating machines as well as NDT applications [2]. We propose a new thin shell magnetic and conducting finite element compatible with the A-φ and T–Ω formulations associated to the Whitney's element.

II. The continuous model of thin plate

The magnetic field $\mathbf{H}$ is null-divergence and must verify in the volume of the shell:

$$\Delta\mathbf{H} - i\omega\mu\sigma\mathbf{H} = 0 . \qquad (1)$$

For a thin volume with neutral surface S, currently two hypothesis are made : first, the "flat plate" allows to neglect curvatures in laplacian development of (1) in local coordinates (x, y, z) and to split the variations of $\mathbf{H}$ on S noted $\mathbf{H}_S(x, y)$ and in depth noted $\alpha(z)$:

$$\mathbf{H}(x, y, z) = \alpha(z)\mathbf{H}_S(x, y) . \qquad (2)$$

Second, the "surface invariant" hypothesis consists in neglecting the skin depth $\delta = (\omega\mu\sigma)^{-0.5}$ reported to the caracteristic length of $\mathbf{H}_S$ on S, which allows analytical integration of (1) in the depth:

$$\mathbf{H}(x, y, z) = \alpha^-(z)\mathbf{H}_S^-(x, y) + \alpha^+(z)\mathbf{H}_S^+(x, y) . \qquad (3)$$

where $\alpha^-(z)$ and $\alpha^+(z)$ are hyperbolic shape functions easy to calculate, $\mathbf{H}_S^-$ and $\mathbf{H}_S^+$ are values of $\mathbf{H}_S^-$ on faces.

Note that an important consequence of the invariant hypothesis is the normal component of any vector field solution of (1) is small compared to the tangential one. Besides, the thickness of the plate $2e$ only impacts the frequencial behavior: if $e \ll \delta$, a so-called "thin layer" is obtained, if $e \gg \delta$, a "recto-verso impedance" results.

It will be efficient to reduce the continuous model (3) to the following integral identity:

$$\int_{z=-e}^{z=+e}\left((\partial_z\mathbf{H})^2 + i\omega\sigma\mu\mathbf{H}^2\right)dz = i\omega\sigma\mu\begin{bmatrix}\mathbf{H}^-\\ \mathbf{H}^+\end{bmatrix}^T[\mathbf{C}]\begin{bmatrix}\mathbf{H}^-\\ \mathbf{H}^+\end{bmatrix} . \qquad (4)$$

$[\mathbf{C}]$ is a (6x6) symmetric matrix depending on δ and e.

III. The host finite element formulations

We import the model (3) in both electric and magnetic formulations the finite element domain of which is composed of conducting volumes V_σ bounded by S_σ included in a magnetic volume V_μ bounded by S_μ:

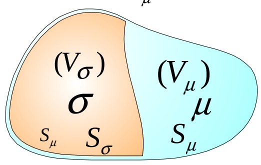

Fig. 1: Magnetic and conducting domains

Note that there is no need to represent the sources in that case and that the electric field outside the conductors is ignored by quasi-static approximation. The variational approach states that:

A. Electric formulation ($A-\varphi$)

The vector potential $\mathbf{A}$ in V_μ such as $\mathbf{B} = \nabla\times\mathbf{A}$ and the scalar potential φ in V_σ such as $\mathbf{E} = -i\omega\mathbf{A} - \nabla\varphi$ minimize the following form with imposed $\mathbf{H}_{S_\mu}\times\mathbf{n}_{S_\mu}$ and $\mathbf{J}_{S_\sigma}\cdot\mathbf{n}_{S_\sigma}$:

$$\begin{aligned}\mathcal{F}(\mathbf{A},\varphi) = &\int_{V_\mu}\frac{1}{\mu}(\nabla\times\mathbf{A})^2\,dV_\mu - 2\int_{S_\mu}\mathbf{A}\cdot\mathbf{H}_{S_\mu}\times\mathbf{n}_{S_\mu}\,dS_\mu\\ &+\frac{1}{i\omega}\left(\int_{V_\sigma}\sigma\left(i\omega\mathbf{A}+\nabla\varphi\right)^2 dV_\sigma + 2\int_{S_\sigma}\varphi\,\mathbf{J}_{S_\sigma}\cdot\mathbf{n}_{S_\sigma}\,dS_\sigma\right)\end{aligned} . \qquad (5)$$

B. Magnetic formulation ($T-\Omega$):

The vector potential $\mathbf{T}$ in V_σ such as $\mathbf{J} = \nabla\times\mathbf{T}$ and the scalar potential Ω in V_μ such as $\mathbf{H} = \mathbf{T} - \nabla\Omega$ minimize the following form, with imposed $\mathbf{E}_{S_\sigma}\times\mathbf{n}_{S_\sigma}$ and $\mathbf{B}_{S_\mu}\cdot\mathbf{n}_{S_\mu}$, and with $\mathbf{T}\times\mathbf{n}_{S_\sigma} = 0$ on isolated boundaries :

$$\begin{aligned}\mathcal{F}(\mathbf{T},\Omega) = &\frac{1}{i\omega}\left(\int_{V_\sigma}\frac{1}{\sigma}(\nabla\times\mathbf{T})^2 dV_\sigma - 2\int_{S_\sigma}\mathbf{T}\cdot\mathbf{E}_{S_\sigma}\times\mathbf{n}_{S_\sigma}\,dS_\sigma\right)\\ &+\int_{V_\mu}\mu\left(\mathbf{T}-\nabla\Omega\right)^2 dV_\mu + 2\int_{S_\mu}\Omega\mathbf{B}_{S_\mu}\cdot\mathbf{n}_{S_\mu}\,dS_\mu\end{aligned} . \qquad (6)$$

Note that the surface integrals vanish in case of Dirichlet condition, i.e., $\mathbf{A}\times\mathbf{n}_{S_\mu}$, $\mathbf{T}\times\mathbf{n}_{S_\mu}$, φ or Ω imposed, null or not.

C. The shell element

μ_C and σ_C are the properties of the shell considered as a thin volume in the magnetic domain:

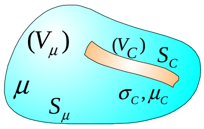

Fig. 2: Volume shell in the magnetic domain

The electromagnetic shell functional is the same as above, excepting that fields can be identified (in V_C only) to their tangent component and can be written $\mathbf{E} \simeq \mathbf{e}$ and $\mathbf{H} \simeq \mathbf{h}$:

$$\mathcal{F}_C = \int_{V_C} \frac{1}{i\omega}\sigma_C \mathbf{e}^2 + \mu_C \mathbf{h}^2 dV_C \tag{7}$$

Condensing directly (7) through the depth z thanks to (4), the functional becomes, assuming the plane hypothesis:

- in the electric form : $[\![\varphi_S]\!] = 0$ and $\mathbf{j} \times \mathbf{n}_S \simeq \partial_z \mathbf{h}$:

$$\mathcal{F}_C(\mathbf{a},\varphi) = \frac{1}{i\omega}\int_{S_C} \sigma_C \left(i\omega \begin{bmatrix} \mathbf{a}^- \\ \mathbf{a}^+ \end{bmatrix} + \nabla_S \varphi \right)^T [\mathbf{C}] \left(i\omega \begin{bmatrix} \mathbf{a}^- \\ \mathbf{a}^+ \end{bmatrix} + \nabla_S \varphi \right) dS_C \tag{8}$$

- in the magnetic form : $i\omega\mu_C\sigma_C\, \mathbf{n}_S \times \mathbf{h} \simeq \partial_z \mathbf{j}$:

$$\mathcal{F}_C(\Omega) = \int_{S_C} \mu_C \begin{bmatrix} \nabla_S \Omega^- \\ \nabla_S \Omega^+ \end{bmatrix}^T [\mathbf{C}] \begin{bmatrix} \nabla_S \Omega^- \\ \nabla_S \Omega^+ \end{bmatrix} dS_C \tag{9}$$

Matrix [C] is coming from (4) and $\mathbf{a}^-$, $\mathbf{a}^+$, Ω^-, Ω^+ are vector and scalar magnetic potentials of both faces of V_c.

Since unknowns are compatible, plugging (8) into (5) and (9) into (6) can be done using a simple assembly procedure.

IV. THE CONTINUITY AT INTERFACES

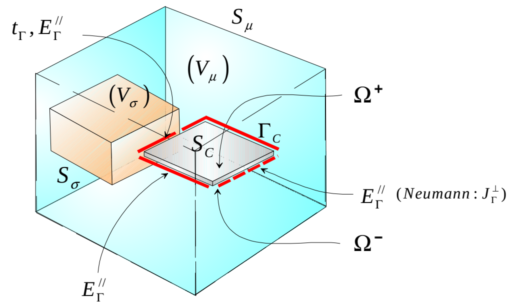

Fig.3: Sample coupled system for edge continuity testing (magnetic case)

Nevertheless, the structure of surface terms of (5) and (6) governs the kind of continuity at interfaces of the shell: on the faces, the continuity is strong for the tangent magnetic field and the normal current in electric form. It is the same for the tangent electric field and the normal induction in magnetic form. On the edge surface which is not discretised in depth, a lack of strong continuity exists, and as the tangential and normal components are permuted, the normal current in magnetic form and the normal induction in electric form are relaxed. The continuity can be restored via Lagrange multipliers which lead, in magnetic case for example, to a "belt element" (see Fig.3) preventing current leakage through isolated sections and ensuring the flux transfer on connected one.

The implementation choice of treating the element as a true surface (supporting $\mathbf{a}$ or Ω jumps) or as a thin volume (connected to others like in the test below) is totally free

V. FIRST ELEMENTARY 2D-TEST

The shell element was tested in $(A-\varphi)$ electric form to evaluate the impact of a thin copper deposit on the external wall of a steel GV tube. Both exact and 2D-approached geometry are shown on Fig. 4. The conductivity ratio is 50 and a skin depth in copper is taken around $100\mu m$.

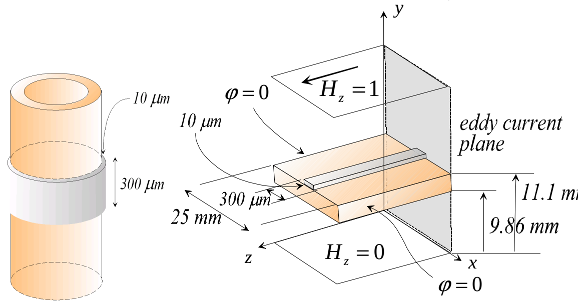

Fig. 4. : Copper deposit on a GV tube and his equivalent 2D model

Fig. 5 shows the distorsion of current (or iso-induction) axial lines and of iso-potential cross-sections in the vicinity to the deposit. A comparison of induction computed with an "exact" refined mesh (10x30 elements), a "coarse" mesh (1x30 elements) and a "shell" mesh (30 shells) proves that the average relative error drops from 0,01 down to 0,001 when the coarse mesh is replaced by the shell mesh.

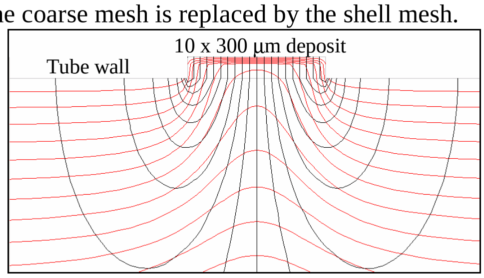

Fig. 5: Perturbation of eddy currents due to a 10 x 300 µm deposit

VI. CONCLUSION

A shell electromagnetic finite element easily mixable with other 3D elements of *Code_Carmel3D* library is defined. A great attention is paid to the continuity on edges so that the coupled model can cope not only with standard shielding problems, but also, after some tests, with circuit-shell or shell-shell coupled systems (as faulty isolated layers in electrical machine magnetic cores).

VII. REFERENCES

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